Crossing of Specific Heat Curves in Liquid Helium 3 and Heavy Fermion Systems

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Specific heat curves for various pressures, in many correlated electron systems, have been seen to cross at a point. We analyze this behavior using the spin fluctuation theory. It is found that when the system is considered near a ferromagnetic instability, the curves cross at a point and for systems near an antiferromagnetic instability, they cross at two points. The crossing behavior is related to crossover of these systems from quantum to classical fluctuation regimes. For detailed analysis, a weak linear pressure dependence of the crossover scale $\alpha(0)T_F$ is assumed.

Helium-3 is a Fermi system with a degeneracy temperature ~ 5 K. However, in the normal phase it behaves like a dense classical liquid for T $\geq .5$ K and like a degenerate Fermi liquid below 0.2 K. Moreover, the (nuclear) spin susceptibility of ^{3}He varies between 10 - 25 times the free Fermi gas susceptibility, χ_P, depending on pressure. The coefficient γ of the linear term in specific heat is also large. There have been theoretical attempts to explain the low temperature behavior of ^{3}He from two points of view phenomenologically. According to the first point of view, known as the spin fluctuation theory, the liquid is regarded as if it is near a ferromagnetic instability because of the largeness of χ. In this theory the temperature variation of various physical quantities is governed by transverse and longitudinal spin fluctuations. Though the actual transition does not take place, the effect of fluctuations is observable over a wide temperature range at low temperatures. This theory explains the temperature variation of many physical properties like spin susceptibility, specific heat quantitatively over a wide temperature range.[1,2] In the second point of view one considers ^{3}He as a liquid in the vicinity of a liquid to solid or the Mott-Hubbard "metal-insulator" transition. The reason being that the velocity of sound is large in liquid ^{3}He. The compressibility at low temperatures is almost same as the compressibility of the solid phase. The liquid becomes sluggish at low temperatures before finally becoming a superfluid. This approach has been successful for understanding the pressure dependence of many properties.[3]

In a recent publication[4] Vollhardt has drawn attention to intersection at a point of specific heat $C_V(T)$ curves for liquid helium 3 and other correlated Fermi systems. In the present work we want to emphasize that this behavior, particularly for ^{3}He, can be understood within the spin fluctuation theory. The formalism has been developed in detail earlier. In the following we use some results from[1,2,5] to discuss the crossing point in the specific heat curves.

The spin fluctuation contribution to the free energy within the mean fluctuation field approximation (or quasi harmonic approximation) is given by,[2]

$$\Delta\Omega = \frac{3T}{2}\sum_{q,m}\ln\{1 - U\chi^0_{qm} + \lambda T\sum_{q',m'}D_{q'm'}\}. \quad (1)$$

Where $D_{q,m}$ is the fluctuation propagator which is related to inverse dynamical susceptibility, χ^0_{qm} is the free Fermi gas (Lindhardt) response function, and λ is the fluctuation coupling constant. The argument of the logarithm is related to inverse dynamic susceptibility. Considering only the thermal part of the integral and ignoring the zero point part, we perform the frequency summation and obtain,

$$\Delta\Omega_{Thermal} = \frac{3}{\pi}\sum_q\int_0^\infty \frac{d\omega}{e^{\omega/T}-1}\arctan\{\frac{\pi\omega/4q}{\alpha(T)+\delta q^2}\},$$

$$(2)$$

Integrating over frequency, we get,

$$\Delta\Omega_{Thermal} = 3T\sum_q\left(\ln\Gamma(y) - (y - \frac{1}{2})\ln(y) + y - \frac{1}{2}\ln(2\pi)\right).$$

$$\text{(3)}$$

where, $y = q(\alpha(\tau) + \delta q^2)/(\pi^2\gamma\tau)$ with $\gamma = 1/2$, $\delta = 1/12$, $\tau = T/T_F$, the wavevector q is given in units of Fermi momentum k_F and the energy is in units of Fermi energy. Once the free energy correction is known, the specific heat correction is given by

$$\frac{\Delta C_v}{k_B} = -T\frac{\partial^2\Delta\Omega}{\partial T^2}$$

$$= -3T^2\sum_q\left[(\frac{2}{T}\frac{\partial y}{\partial T} + \frac{\partial^2 y}{\partial T^2})\phi(y) + (\frac{\partial y}{\partial T})^2\frac{\partial\phi(y)}{\partial y}\right]$$

$$= 6\int q^2 dq\{\phi'(y)(\frac{q}{\pi^2\gamma}\frac{\partial\alpha(T)}{\partial T} - y)^2$$

$$+ T\phi(y)\frac{q}{\pi^2\gamma}\frac{\partial^2\alpha(T)}{\partial T^2}\}.$$

$$\text{(4)}$$

The function $\phi(y)$ is related to the fluctuation self energy summed over frequency. It varies as $1/2y$ for $y \ll 1$ and as $1/12y^2$ for $y \gg 1$.

Clearly the calculation of specific heat correction involves the temperature dependence of spin susceptibility. A self consistent equation for the temperature dependence of the inverse susceptibility (in units of χ_P) within one spin fluctuation approximation is given by,[1,5]

$$\alpha(\tau) = \alpha(0) + \frac{\lambda}{\pi}\sum_q q\phi(y) \qquad (5)$$

For a finite $\alpha(0)$ there are two regions of temperature[1]. For $\tau < \alpha(0)$, which corresponds to $y \gg 1$, one gets an enhanced Pauli susceptibility with standard paramagnon theory corrections, $\alpha(\tau) = \alpha(0) + x\tau^2/\alpha(0)$, where x turns out to be ≈ 0.44. For $\alpha(0) < \tau < 1$, $\alpha(\tau) \sim \tau^n$ with the exponent $1 \leq n \leq 4/3$. This result for the susceptibility in this regime mimics the classical Curie Weiss susceptibility. Hence even for a Fermi system for $T < T_F$, the susceptibility behaves like the one for a collection of classical spins. This behavior agrees well[1] with experimental results of Thompson et. al.[6].

In the low temperature limit $(y \gg 1)$ it turns out that α' and α'' do not contribute to the leading temperatutre dependence. The specific heat correction is given by,

$$\frac{\Delta C_v}{k_B} = -\sum_q\frac{\pi^2\tau}{4q(\alpha + \delta q^2)}. \qquad (6)$$

The phase space integral reproduces the standard paramagnon mass enhancement result, $\tau\ln\alpha$ for ΔC_v. The higher order terms give a $(\tau^3/\alpha(0)^3)\ln(\alpha(0)/\tau^2)$.

In the classical regime, $\alpha(0) \leq \tau \ll 1$ the small y approximation holds, and $\alpha(\tau)$ varies as τ. In Eq. 4 $\alpha'' = 0$ and $\alpha' = \alpha/\tau$, leading to ΔC_v falling as $1/\tau^2$ at higher temperatures.

The result is that similar to the susceptibility variation there are two regimes for the specific heat also and the behavior of the specific heat in these two regimes is qualitatively different. At low temperature there is an enhanced linear rise leading to a peak around 0.15 K and thereafter a slow fall as the temperature increases. This peak marks a transition from quantum to classical spin fluctuation regimes. Fig.1 shows a set of curves of $(C_v(P,T) - C_v(0,T))/C_v(0,T)$ for P = 15 Bar to P = 30 Bar. To calculate the specific heat, the free electron part $(\pi^2 T/2T_F)$ has been added to $\Delta C_v(T)$. The value of $\alpha(\tau)$ has been calculated self consistently using Eqn.5 and then used as an input in the specific heat calculation. The coupling constant λ has been chosen to be 0.08 and the cutoff for the momentum sum, $1.2k_F$. The crossover temperature is related to $\alpha(0)T_F$ which depends on pressure.
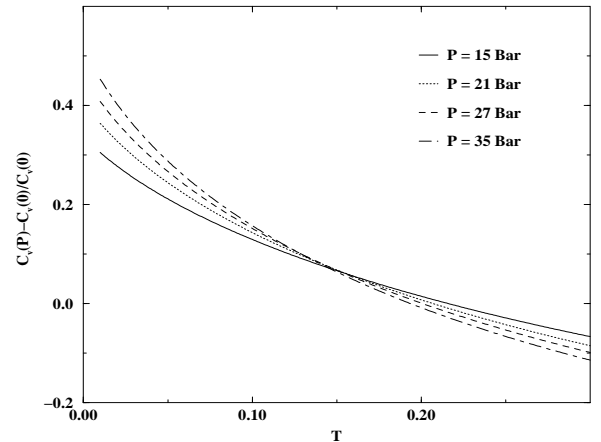
FIG. 1. $(C_v(P,T) - C_v(0,T))/C_v(0,T)$ as a function of T for P = 15, 21, 27 and 34.36 Bar respectively. The values of coupling constant λ and momentum cutoff are chosen to be 0.08 and $1.2k_F$ respectively. The curves for pressures below 15 Bar cross slightly away from the point shown in the curve for reasons mentioned in the text below.
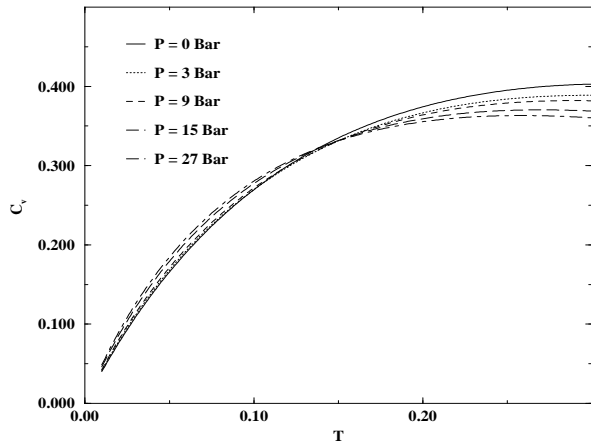

FIG. 2. $(C_v(P,T)$ as a function of T for P = 0, 3, 15, and 27 Bar respectively. The values of coupling constant λ and momentum cutoff are chosen to be 0.08 and $1.2k_F$ respectively.

It turns out that for $\alpha(0)T_F$ scaling linearly with pressure the specific heat curves for various pressures cross at a point. The linear scaling is experimentally observed above pressures about 15 kbar. However, at small pressures there is some departure. In Fig.2, $C_v(P,T)$ is plotted as a function of T for various values of P, assuming a linear dependence of $\alpha(0)T_F$ on pressure. It is very clear from the figure that for linear scaling of $\alpha(0)T_F$ the curves cross at a point. The crossing point increases very slightly with increase in cutoff and with decrease in λ but the nature of crossing is not affected. The crossing of the specific heat curves at various pressures has also been discussed in the earlier work of Seilers et. al.[7] but they do not match with Greywall's experimental findings

of crossing at a point[8], in fact there is wide range of temperatures over wich the curves cross. In the present spin fluctuation calculation with the assumption of a linear scalling the crossing occurs at a point.

We have used the terms quantum and classical in the discussion above, because, temperatures below $\alpha(0)T_F$ essentially define a regime where one gets a Fermi liquid behavior whereas at the higher temperatures, fluctuations get correlated resulting in the classical behavior for susceptibility and the specific heat corrections. The distinction, quantum versus classical, becomes clear when one takes the limit $\alpha(0) \to 0$. In that case the Curie law is obtained down to zero degree,[5] while in the opposite limit $(\alpha(0) \to 1)$ one gets the Pauli susceptibility. The crossing temperature T_+ is related to the pressure derivative of this this crossover scale. This crossover characterizes the change in temperature dependence of all thermodynamic properties.
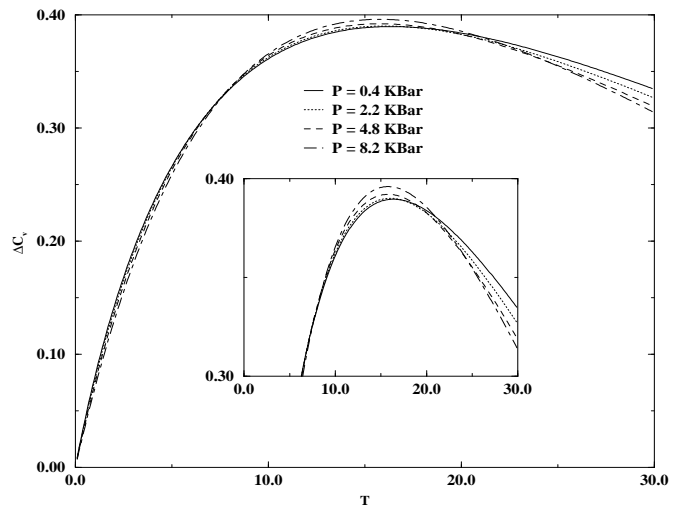

FIG. 3. $\Delta C_v(P,T)$ as a function of T for P = 0.4, 2.2, 4.8 and 8.2 KBar respectively. The values of coupling constant λ and momentum cutoff are chosen to be 1.5 and $1.2k_F$ respectively. Inset figure shows the region close to the crossover points.

There are some heavy fermion materials, for example $CeCu_{6-x}Au_x$[9], $CeAl_3$[10] in which the specific heat curves

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cross. However, the crossing occurs at two points, in case of $CeAl_3$ these temperatures are 5K and 17K respectively. It is possible to cast the behavior of these materials in terms of spin fluctuation theory for antiferromagnets. We have calculated the specific heat corrections by writing the equations for the susceptibility enhancement and specific heat near an antiferromagnetic instability.[5] The curves do cross at two points as shown in the Fig3. The detailed comparison with experiments is in progress and will be presented elsewhere.

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